88087203

International Baccalaureate®
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MATHEMATICS
HIGHER LEVEL
PAPER 3 – DISCRETE MATHEMATICS

Thursday 13 November 2008 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 19]*

(a) Convert the decimal number 51966 to base 16. *[4 marks]*

(b) (i) Using the Euclidean algorithm, find the greatest common divisor, d, of 901 and 612.

(ii) Find integers p and q such that $901p + 612q = d$.

(iii) Find the least possible positive integers s and t such that $901s - 612t = 85$. *[10 marks]*

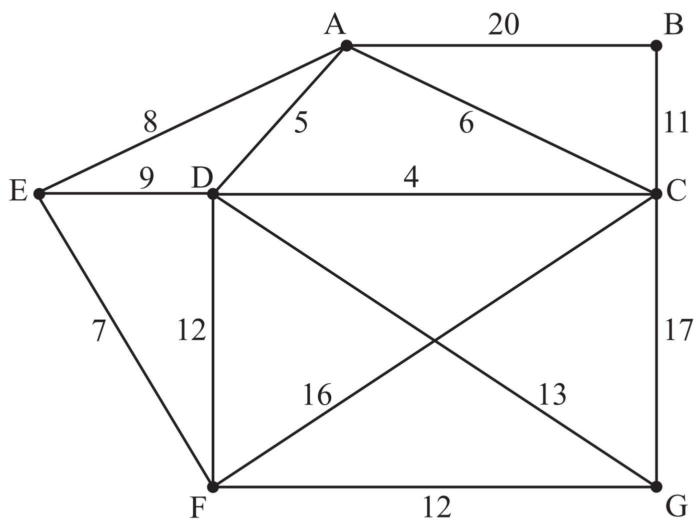
(c) In each of the following cases find the solutions, if any, of the given linear congruence.

(i) $9x \equiv 3 \pmod{18}$

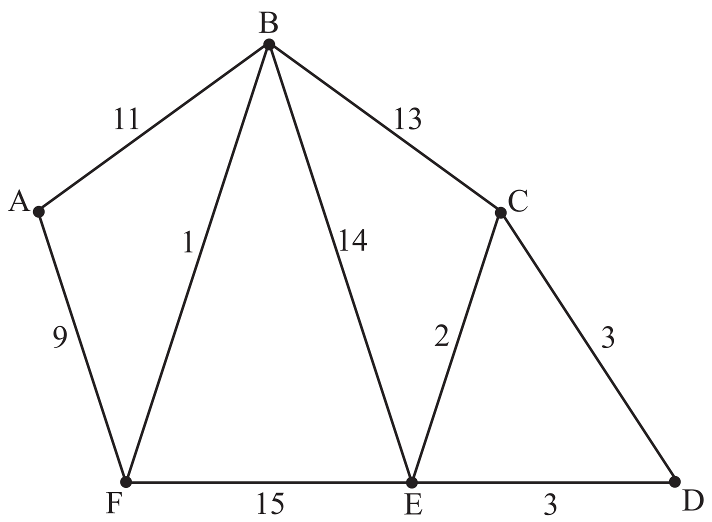
(ii) $9x \equiv 3 \pmod{15}$ *[5 marks]*

2. *[Maximum mark: 12]*

(a) Use Kruskal's algorithm to find the minimum spanning tree for the following weighted graph and state its length. *[5 marks]*



(b) Use Dijkstra's algorithm to find the shortest path from A to D in the following weighted graph and state its length. *[7 marks]*



3. *[Maximum mark: 12]*

(a) Write 457 128 as a product of primes. *[4 marks]*

(b) Numbers of the form $F_n = 2^{2^n} + 1,\ n \in \mathbb{N}$ are called Fermat numbers.

Find the smallest value of n for which the corresponding Fermat number has more than a million digits. *[4 marks]*

(c) Prove that $22 \mid 5^{11} + 17^{11}$. *[4 marks]*

4. *[Maximum mark: 17]*

(a) A connected planar graph G has e edges and v vertices.

(i) Prove that $e \ge v - 1$.

(ii) Prove that $e = v - 1$ if and only if G is a tree. *[4 marks]*

(b) A tree has k vertices of degree 1, two of degree 2, one of degree 3 and one of degree 4. Determine k and hence draw a tree that satisfies these conditions. *[6 marks]*

(c) The graph H has the adjacency matrix given below.

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

(i) Explain why H cannot be a tree.

(ii) Draw the graph of H. *[3 marks]*

(d) Prove that a tree is a bipartite graph. *[4 marks]*